# ON THE VARIABILITY OF THE EXPONENT IN REAL-GAS PROCESSES

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The problem of an equivalent constant exponent in real-gas processes is analyzed here. A numerical example is shown where air undergoes adiabatic compression.

In the derivation of the equations for a real gas there appear differential expressions of the form

$$\frac{dp}{p} + \varkappa(p, T)\frac{dv}{v} = 0, \tag{1}$$

where the coefficient $\varkappa$ is a variable quantity which, generally, depends on the pressure and the temperature, making the integration of the equation more difficult. In [1] expression (1) was integrated on the assumption that the coefficient is much more nearly constant than c, p, v, and T. In [2-4] this expression was integrated with $\varkappa$ = const, whereupon the process was calculated by the resulting equation with the exponent $\varkappa$ constant and equal to the average between its actual initial and final values in the process. The validity of such an approach was proved there by the calculation of several processes.

Both approaches are not rigorous and, therefore, it would be of interest to analyze Eq. (1) with a variable coefficient $\varkappa$. Obviously, for a given process there exists a unique relation between coefficient $\varkappa$ and the specific volume. One may, therefore, consider the variables in (1) to be separated and this equation to be integrable by parts:

$$\ln p + \varkappa \ln v - \int \ln v\, d\varkappa = \ln C, \tag{2}$$

from where

$$pv^{\varkappa} \exp\left(-\int \ln v\, d\varkappa\right) = p_1 v_1^{\varkappa_1}. \tag{3}$$

It follows from (2) and (3) that the end parameters of a process depend not only on the value of the exponent at the start and at the end of the process but also on the path through which this exponent has varied. The equivalent constant value of the exponent in the equation which relates the start and the end points of a real process 1-2 can be defined by the following expression:

$$\bar{k} = \frac{\varkappa_1 \ln v_1 - \varkappa_2 \ln v_2 + \int\limits_1^2 \ln v\, d\varkappa}{\ln (v_1/v_2)}.$$

For a specific process, the function $\varkappa = \varkappa(v)$ may be approximated by a polynomial $\varkappa = a + bv + cv^2 + dv^3 + \ldots$, and an integration of the equation

$$\frac{dp}{p} + (a + bv + cv^2 + dv^3 + \ldots)\frac{dv}{v} = 0$$

will then yield

$$\ln p + a \ln v + bv + \frac{c}{2} v^2 + \frac{d}{3} v^3 + \ldots = \text{const}. \tag{4}$$

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The equation describing the process 1-2 is of the form

$$p_2 v_2^a = p_1 v_1^a \exp\left[b(v_1 - v_2) + \frac{c}{2}(v_1^2 - v_2^2) + \frac{d}{3}(v_1^3 - v_2^3) + \ldots\right],$$

i.e., the process equation in its final form is a product of a power function and an exponential function. From (4) one can derive the equivalent constant value of the exponent:

$$\bar{k} = a + \frac{b(v_1 - v_2) + \frac{c}{2}(v_1^2 - v_2^2) + \frac{d}{3}(v_1^3 - v_2^3) + \ldots}{\ln(v_1/v_2)}. \tag{5}$$

In an adiabatic compression of air from $p_1 = 450$ bar and $T_1 = 475$°K to $p_2 = 750$ bar, the exponent of the adiabatic may be expressed as the following function of the specific volume*: $\varkappa = 4.1284 - 916.7\,v + 95820\,v^2$ with $\varkappa_1 = 2.0387$ and $\varkappa_2 = 2.2568$. The value of the equivalent constant exponent calculates to be $\bar{k} = 2.1324$.

The average exponent of the adiabatic is

$$k_{av} = \frac{2.0387 + 2.2568}{2} = 2.1477.$$

For a check, let us calculate the exponent from the final process parameters:

$$\bar{k} = \frac{\lg(750/450)}{\lg(0.003755/0.002957)} = 2.1370.$$

It is noteworthy that while, in general, Eq. (1) must be supplemented by an equation which relates the temperature to the pressure or to the specific volume and the complete system of equations must then be considered, the analytical relation $\varkappa(p, T)$ is very unwieldy and cannot always be established and, therefore, our method seems simpler.

## NOTATION

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| p | is the pressure; |
| T | is the absolute temperature; |
| v | is the specific volume; |
| $\varkappa$, k | are the adiabatic exponents. |

## LITERATURE CITED

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*This function has been obtained by approximating several values of the adiabatic exponent calculated by a numerical method with the aid of tables in [5].